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PARAMETER ESTIMATION IN CONTINUOUS TIME DOMAIN

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Abstract

This paper will aim to presents the applications of a continuous-time parameter estimation method for estimating structural parameters of a real bridge structure. For the purpose of illustrating this method two case studies of a bridge pile located in a highly seismic risk area are considered, for which the structural parameters for the mass, damping and stiffness are estimated. The estimation process is followed by the validation of the analytical results and comparison with them to the measurement data. Further benefits and applications for the continuous-time parameter estimation method in civil engineering are presented in the final part of this paper.

Key words: parameter estimation, structural health monitoring, system identification.

1. Introduction

Nowadays, most of the bridges build in Romania have exceeded their designed life span and are experiencing a loss in performance due to aging and due to the presence of aggressive agents. This decrease in performance of the constitutive materials has a direct effect on the serviceability of the bridge, [1]. Bridges are part of a transportation infrastructure network and play a major role in economic development and thus maintaining these structures safe and reliable for everyday use is very important, [2]. When a bridge becomes unavailable due to maintenance and /or repair actions this can lead to negative social and economic effects, [3].

The importance of assessing or evaluating the condition of existing bridges can be justified by the following main reasons, [1]:

1.The increase in traffic densities that make traffic loads much greater than those for which the bridge was designed for;

2. The deterioration or damage of the bridge structure can lead to a decrease its strength;

3.Changes in design codes that have reduced the safety levels.

The lack of a reliable maintenance framework for bridges has contributed to the deterioration of these structures over their years in service. In Europe a number of studies have been performed in order to collected data and check if the analytical and numerical models can correctly represent the structural behavior of existing damaged bridges, [1].

Early damage detection has direct implications in safety maintenance and keeping the bridges reliable for daily. Most of the current damage detection methods rely on visual or localized experimental methods, [4]. The inspections carried out on bridges often interfere with their operational conditions, [2] and they require that the location of the damage to be known a priori. Due to these limitations, these methods detect the damage on or near the surface of the bridge.

The use of non-destructive test data in structural health monitoring SHM is regarded as an important field of study in model updating, structural evaluation and damage assessment, [5].

Non-destructive techniques NDT can be used as a mean to inspect bridge structures without disrupting or impairing its serviceability. The NDT techniques are based on based on comparing and analyzing the properties of the materials of the bridge and can be used to interpret the structural condition of the bridge by observing the change in its global behavior. One

*Member correspondent of Romanian Academy of Technical Sciences ASTR method by which this can be achieved is by means of vibration test data, collected from sensor measurements, [6]. The vibration measurements can help identify changes in local stiffness, mass and damping and by observing these changes one can predict the response of the bridge in correlation with the measured data. The detected changes in the structural parameters are correlated with structural damage.

2. Parameter identification in continuous-time domain

System identification Sys-Id can be used in SHM of bridge structures as the process of finding a model based on dynamic input and output measurements. Some of the deficiencies present in the methodology of bridge health assessment in Romania could be resolved by applying the concepts of Sys-Id in SHM of road bridges with the help of mobile laboratories used for collecting data, [6].

A bridge system can be represented by means of a mathematical model described by either a differential equation system expressed in continuous-time, or either by different equation systems expressed in discrete-time respectively. It is important to development and use of an a priori model and this can be achieved by calibrating the models parameters, such as the stiffness, damping or modal parameters, in order to have a minimal difference between the initial model and the measured results, [7].

The dynamic behavior of a bridge structure can be described by a reduced model of the bridge. The most common used model in Sys-Id is the lumped parameter model, due to its simplicity. A n-degree of freedom structure can be described by the following linear differential equation, [8]:

$$\mathbf{M}\cdot\ddot{\mathbf{Z}}(\mathbf{t})+\mathbf{C}\cdot\dot{\mathbf{Z}}(\mathbf{t})+\mathbf{K}\cdot\mathbf{Z}(\mathbf{t})=\mathbf{F}(\mathbf{t}) \qquad (1)$$

where **M**, **C** and **K** are the mass, damping and stiff-ness matrices, of dimension n × n, **Z(t)** and **F(t)** are the displacement and force vectors respectively of dimension n × 1.

Depending on the type of processes they describe a dynamic system model can either be a continuous-time or a discrete-time process. The continuous-time models are described using derivatives or integrals while the discrete-time models are represented using algebraic approximations, [9].

By means of continuous-time models one can determine the physical parameters of a structure. A continuous time-model can be obtained either using an indirect approach or either by means of a direct approach. The indirect approach uses experimental measured data to estimate a discrete-time model and this model will be later transferred into a continuous-time model. Using the direct approach one can determine a continuous-time model using discrete-time data. The differential equations used to describe the system using the direct approach are represented in a linear regression form, as expressed by Eq (2). Discretization of a continuous-time model This process of approximation by discretization of a continuous-time model eliminates the complex continuous–time calculus, [10].

$$\frac{d^2z_n(t)}{dt^2}=\varphi_n^T(t)\theta_n+\varepsilon_n(t),\quad q=n \qquad (2)$$

where

$$\theta_n=\begin{bmatrix} a_{nn-1,2} & a_{nn,2} & a_{nn-1,1} & a_{nn-1,1} & b_n \end{bmatrix}^T \quad (3)$$

and

$$\varphi_n(t)=\begin{bmatrix} \frac{dz_{n-1}(t)}{dt} & \frac{dz_n(t)}{dt} & z_{n-1}(t) & z_n(t) & F_n(t) \end{bmatrix}^T \quad (4)$$

where **a** and **b** are parameters of the model, **z(t)** and **F(t)** are the displacement and force vectors respectively.

3. Poisson moment functional description

The coefficients of the continuous-time lumped model can be estimated by pre-filtering the input data using a Linear Dynamical Operations LDO. By applying a LDO to Eq. (2) one obtains:

$$LDO\left\{\frac{d^2z_n(t)}{dt^2}\right\}=LDO\left\{\varphi_n^T(t)\right\}\theta_n+LDO\left\{\varepsilon_n(t)\right\} \qquad (5)$$

The Poisson Moment Functional PMF is a LDO that can help solve the derivative terms of the differential equations. This pre-filter can be used to determine the continuous-time parameters by using traditional parameter estimation methods such as Least Squares LS, Instrumental Variable IV etc., [11]. The PMF of k-order is represented by Eq. (6):

$$LDO\{\circ\}=M_k\{\circ\}\big|_{t=t_f}\overset{def}{=}\int_{t_0}^{t_f}h_{k+1}(\lambda,t_f-t)\cdot\circ dt \quad (6)$$

where Mk is the PMF of k-order; h_{k+1}, gamma kernel of k+1 order; λ, real pole value of the PMF; tf, final time; t, time.

By applying the PMF pre-filtering approach to Eq. (5) one obtains:

$$M_2\left\{\frac{d^2z_n(t)}{dt^2}\right\}=M_2\left\{\varphi_n^T(t)\right\}\theta_n+M_2\left\{\varepsilon_n(t)\right\} \quad (7)$$

where

$$M_2\left\{\frac{d^2z_n(t)}{dt^2}\right\}\bigg|_{t_k}=z_{n\ 2}^{(2)}(t_k) \qquad (8)$$

and

$$M_2\{\varphi_n(t)\}\Big|_{t_k} = [z^{(1)}_{n-12}(t_k) \quad z^{(1)}_{n\;2}(t_k) \quad z^{(0)}_{n-12}(t_k) \quad z^{(0)}_{n\;2}(t_k) \quad F^{(0)}_{n\;2}(t_k)]^T \tag{9}$$

Using the PMF method the differential equations that represent the bridge system, as expressed by Eq. (2) are reduced to an the algebraic form as shown in Eq. (7)- (9).

4. Parameter estimation of a bridge pile using the PMF approach

The PMF approach was used to estimate the structural parameters of a real size pile for a concrete road bridge. The chosen bridge, is locate in Iași municipality, in the Tudor Vladimirescu residential neighborhood. The reinforced concrete bridge consists of a single span superstructure, with a total length of 46 m. The superstructure is made up of concrete box girders that are joined together, with a total length of 13.2 m, that support 3 lanes of traffic. The superstructure is supported by concrete wall-type piles with a total height of 4 m, as shown in Fig. 2.

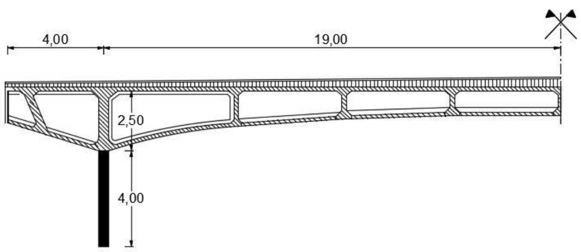

Fig. 2. Longitudinal cross-section of the reinforced-concrete Tudor Vladimirescu bridge, Iași.

The concrete used for the construction of the wall-type piles is of C 30/37 grade, and its characteristics are presented in Table 1.

Table 1 Material characteristics for concrete C 30/37

Weight, ρ	Modulus of Elasticity, E	Moment of Inertia, I
[kg/m^3]	[N/m^2]	[m^4]
2,548.5	33×10^9	0.0704

The analyzed wall-type pile was reduced to a lumped parametric model with 3-DOFs, as depicted in Fig. 2, and its structural parameters were estimated in continuous-time domain.

The benchmark structural parameters for the wall-type pile are presented in Table 2.

Table 2 Structural characteristics for the 3-DOF wall-type pile

Level	Mass [kg]	Damping [Ns/m]	Stiffness [N/m]
1	2.691E+04	5.478E+05	6.970E+09
2	2.691E+04	1.937E+05	8.712E+08
3	3.138E+04	1.871E+05	6.970E+09

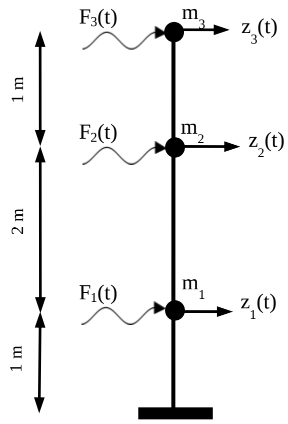

Fig. 1. 3-DOF lumped model of the wall-type pile

For structural parameters of the 3-DOF lumped parametric model were estimated using the PMF approach with the model being subjected to two Load Cases LC. For Load Case 1 LC1, the external forces have been determined using the seismic acceleration response of Vrancea 77 earthquake in EW direction, while the forces in Load Case 2 LC2 have been determined using the seismic acceleration response of the same earthquake but in NS direction.

For LC 1 the input seismic force response is illustrated in Fig. 3

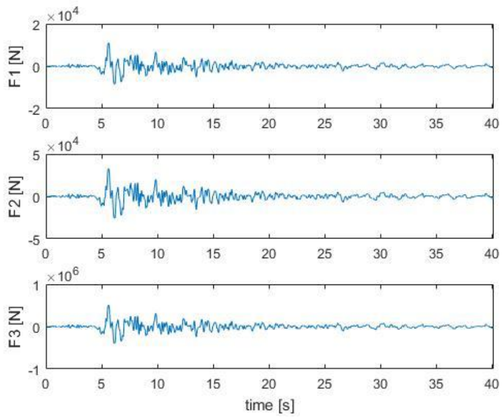

Fig. 3 Force Response for LC1 case study

For LC 1 the 3 DOF lumped model is subjected to the seismic forces for a period of 40.16 seconds and the resulting displacements are illustrated in Fig. 4.

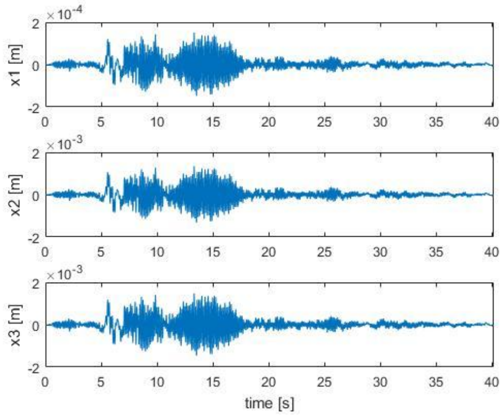

Fig. 4 Displacement responses for LC1 case study

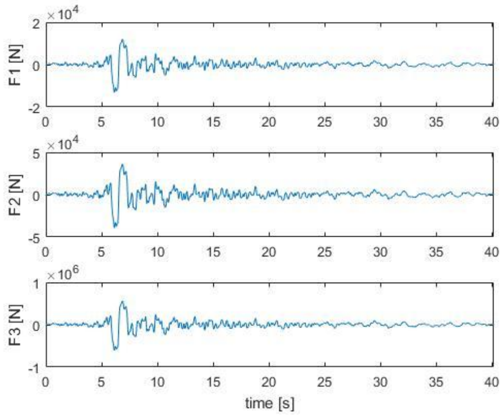

Fig. 5 Responses of forces for LC2 case study

The structural parameters for the mass, damping and stiffness of the 3 DOF model for the wall-type pile were estimated using the LS parameter estimation method, using as input/output data the seismic forces and the resulting displacements respectively. The input and output data used in LC 1 was pre-filtered using the PMF approach and for the estimation process the real pole value for the PMF was chosen to be λ 543 rad/s. The filtered input/output signals were the successfully used to estimate de desired structural parameters in continuous-time domain. Table 3 presents the estimated parameters values for LC1.

Table 3 Estimated parameters based on PMF approach for LC1 case study

Level	Mass [kg]	Damping [Ns/m]	Stiffness [N/m]
1	2.691E+04	5.478E+05	6.970E+09
		1.937E+05	8.712E+08
2	2.691E+04	1.936E+05	8.710E+08
		1.872E+06	6.972E+09
3	3.138E+05	1.871E+06	6.969E+09

For Load Case 2 LC2, the wall-type pile system is excited by the seismic force corresponding to NS direction, and the input forces corresponding to each mass can be visualized in Fig. 5.

The displacement response of the 3 DOF lumped model is determined by applying the seismic forces from LC2 for a period of 40.16 seconds, and can be seen in Fig. 6.

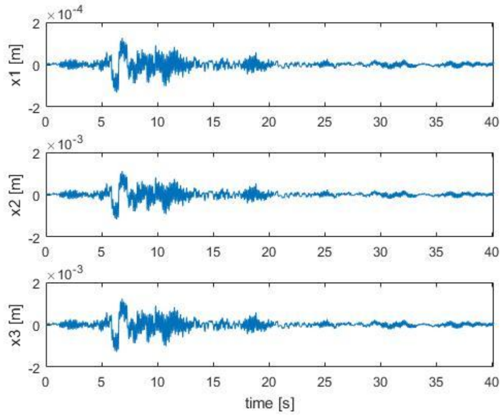

Fig. 6. Displacement responses of the system for LC2 case study

For estimating the structural parameters in LC 2 the chosen real pole value for PMF was λ of 2002 rad/s. The estimate values for the mass, damping and stiffness parameters of the 3-DOF lumped model for LC2 can be seen in Table 4.

Table 4 Estimated parameters based on PMF approach for LC2 case study

Level	Mass [kg]	Damping [Ns/m]	Stiffness [N/m]
1	2.691E+04	5.478E+05	6.970E+09
		1.937E+05	8.712E+08
2	2.691E+04	1.938E+05	8.713E+08
		1.871E+06	6.970E+09
3	3.138E+05	1.871E+06	6.970E+09

For both load cases the estimated parameter values for mass, damping and stiffness using the PMF approach converge to relative close values to the benchmark parameters presented in Table 2.

5. Conclusions

The values for the estimated parameters of the lumped model with 3-DOFs in the continuous-time domain subjected to a seismic excitation force were successfully determined and with a close convergence to the real structural parameter values.

The adopted PMF approach for pre-filtering the input/output signals is a viable method for continuous-time parameter estimation. This approach greatly minimizes the calculus of differential equations, and this is achieved by reducing them to algebraic equations.

This continuous-time models could prove to be useful in the health diagnosis procedure of bridge structures. Changes in parameter values can be detected using the continuous-time model and these can further be used to determine the performance of the overall system. This can have a great impact in the decision making process as it can help maintain a bridges operational function under hazardous seismic events.

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